

Spatial string tension in $N_f = 2$ lattice QCD at finite temperature

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Abstract

The spatial string tension across a crossover from the low temperature phase to the high temperature phase is computed in QCD with two flavors of non-perturbatively improved Wilson fermions at small lattice spacing $a \sim 0.12$ fm. We find that in the low temperature phase spatial string tension agrees well with zero temperature string tension. Furthermore, it does not show increasing for temperatures up to $T = 1.36T_{pc}$, the highest temperature considered. Our results agree with some theoretical predictions.

PACS numbers: 11.15.Ha, 12.38.Aw, 12.38.Gc

I. INTRODUCTION

In view of running and future heavy-ion collisions experiments determination of the (pseudo)critical temperature T_{pc} as well as studies of other features of the finite temperature transition in full QCD with $N_f = 2$ and $N_f = 2 + 1$ flavors of dynamical quarks are subjects of many recent lattice investigations. Three groups employed improved staggered fermions and an improved gauge field action [1, 2, 3]. Simulations with staggered fermions, although computationally less demanding, are made with the use of the fourth root of the fermion determinant. This introduces theoretical uncertainty. Thus studies with the Wilson-type quark actions free of such problem are necessary. The CP-PACS [4], WHOT-QCD [5] and DIK [6] collaborations used improved Wilson fermions with an improved or unimproved gauge field action. In this paper we are using the lattice configurations generated by DIK collaboration.

Recent results [1, 6] show importance of the finite lattice spacing effects in the studies of the finite temperature transition in QCD. The DIK collaboration simulations were done on lattices with large N_t , i.e. the transition was studied at small lattice spacing, close to the continuum limit. For this reason we consider it important to study in more details various features of the finite temperature QCD using gauge field configurations accumulated by DIK collaboration. In this letter we present results for the spatial string tension σ_s .

In the presence of dynamical quarks the flux tube formed between static quark-antiquark pairs breaks at large distances. In contrast the so called spatial Wilson loop, i.e. the Wilson loop with both directions being spatial is found to show non-vanishing spatial string tension even at $T > T_{pc}$. This is called the spatial confinement. σ_s is one of the fundamental quantities to characterize the perturbative and non-perturbative properties of the hot QCD medium. It provides useful information about the effective 3D model, which can be obtained by integrating out heavy modes. It is believed that non-perturbative contributions to observables, which at high temperature are dominated by contributions of magnetic modes, are flavor independent apart from indirect dependence through the flavor dependence of the running coupling. σ_s can be used to test this aspect of the dimensional reduction approach to QCD.

Although quarks are expected to decouple from the spatial observables for $T \gg T_{pc}$ due to dimensional reduction and thus do not affect σ_s in the high temperature limit, it is not

β	κ	a/r_0	$m_\pi r_0$	T/T_{pc}	$\sigma_s a^2$	$\sqrt{\sigma} r_0$
5.2	0.1330	0.2854	2.89	0.78	0.110 (3)	1.16
5.25	0.1335	0.2425	2.98	0.92	0.085 (2)	1.16
5.25	0.1340	0.2272	2.72	0.99	0.069 (4)	1.16
5.25	0.1341	0.2242	2.66	1.01	0.063 (3)	1.15
5.25	0.1342	0.2213	2.60	1.03	0.063 (2)	1.15
5.2	0.1348	0.2192	2.03	1.07	0.058 (2)	1.14
5.2	0.1360	0.1846	1.06	1.36	0.040 (2)	1.13

TABLE I: The parameters of lattice configurations used in computations of σ_s . $\sqrt{\sigma} r_0$ is the physical string tension dependent on $m_\pi r_0$, respective values are taken from [19]

obvious whether the same is true near T_{pc} . Thus lattice computations are also useful to check theoretical predictions for this range of temperatures.

II. DETAILS OF COMPUTATIONS

σ_s was recently computed by RBC-Bielefeld [7, 8] and WHOT-QCD [5] collaborations. Our lattice action is different from that used in [7, 8] and similar to action used in [5]. The important difference from [5] is that we are working at two times smaller lattice spacing, i.e. much closer to the continuum limit. We perform computations on $N_s^3 N_t = 16^3 8$ lattices while in [5] $N_t = 4$ lattices were used. The configurations were generated with the usual Wilson gauge field action and the non-perturbatively $O(a)$ improved Wilson fermionic action S_F :

$$S_F = S_F^{(0)} - \frac{i}{2} \kappa g c_{sw} a^5 \sum_s \bar{\psi}(s) \sigma_{\mu\nu} F_{\mu\nu}(s) \psi(s), \quad (1)$$

where $S_F^{(0)}$ is the original Wilson fermionic action, g is the gauge coupling and $F_{\mu\nu}(x)$ is the field strength tensor. The clover coefficient c_{sw} is determined non-perturbatively [9]. We use configurations generated at two values of the lattice coupling $\beta = 6/g^2$: $\beta = 5.2$ and $\beta = 5.25$, for temperatures between $0.78 T_{pc}$ and $1.36 T_{pc}$. To fix the physical scale and the quark mass results obtained with this action by QCDSF and UKQCD collaborations [10, 11, 12] were used. The scale is fixed by the phenomenological length r_0 [13] and the quark mass is

fixed by $m_\pi r_0$ value. The parameters of our lattices are presented in the TABLE I. Note, that we used for T_{pc} the value depending on $m_\pi r_0$ which was obtained using interpolation of DIK results as function of $m_\pi r_0$ [14]. For this reason values for T/T_{pc} in TABLE I are slightly different from those in [6].

We evaluate σ_s following computational procedure used in [16, 17] where pure gauge theories were investigated. Let us consider z direction as the 'time' direction of the spatial Wilson loops. The temperature dependent spatial static potential $V_s(R)$ is then defined as

$$aV_s(R) = \lim_{Z \rightarrow \infty} \log \frac{W(R, Z)}{W(R, Z + a)}, \quad (2)$$

where $W(R, Z)$ are Wilson loops of size $R/a \times Z/a$ in lattice units. We assume an ansatz for $V_s(R)$ including linear and selfenergy terms:

$$V_s(R) = V_0 + \sigma R. \quad (3)$$

The ambiguous Coulomb term is not included and instead we make our fits at large distances $R/a \geq 5$ to avoid such term contribution. To increase the overlap with the respective ground state the APE smearing [18] of the links orthogonal to this direction was used. Furthermore, the hypercubic blocking (HYP) procedure [15] was applied to links in z -direction to decrease the selfenergy V_0 and thus to decrease the statistical errors. We applied three steps of HYP. It occurred that the first step produced the maximal effect decreasing statistical errors of the potential $V_s(R)$ by factor about 5. Application of the second and third steps of HYP were much less effective but still useful, decreasing statistical errors by about 20%. The limit $Z \rightarrow \infty$ in (2) was approximated by taking the spatial Wilson loops with extension $Z/a = 4$ or 5 to evaluate $V_s(R)$.

III. RESULTS

In Fig. 1 we show the potential $V_s(R)$ with $V_s(r_0)$ subtracted as function of R/r_0 for different temperatures. One can see that potentials for all temperatures agrees with each other quite well, i.e. there is no strong temperature dependence in the temperature interval considered. Respectively the spatial string tension is also virtually independent of temperature as can be seen from Fig. 2. In this figure we present our results for $\sigma_s(T)$ as a dimensionless ratio $\sqrt{\sigma_s(T)}/\sigma$, where σ is the physical string tension at zero temperature [24]. We also

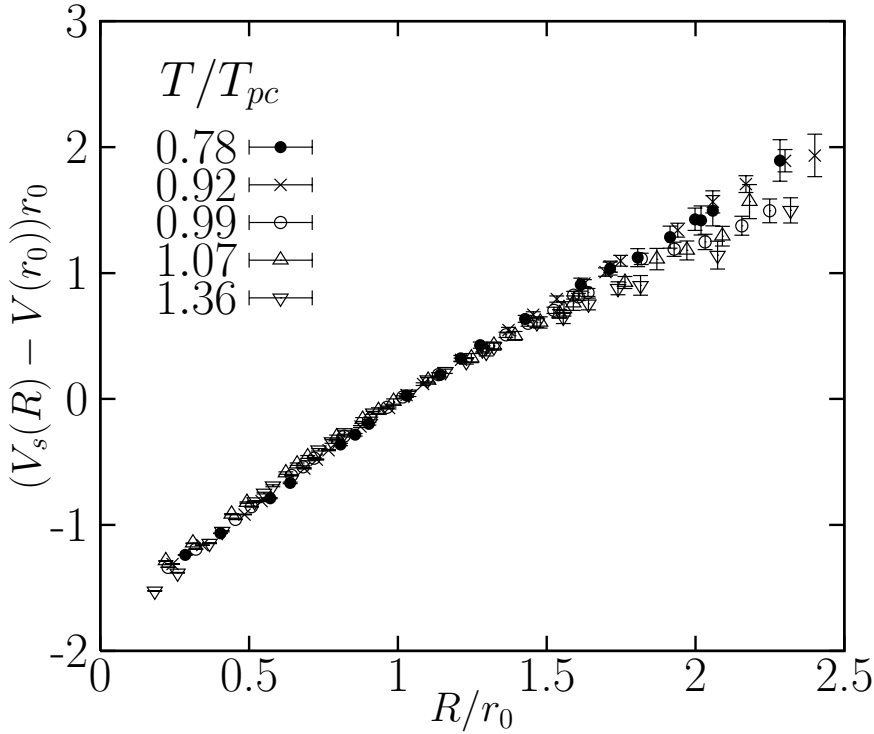


FIG. 1: Spatial static potential $V_s(R)$.

depicted in this figure DIK results for the finite temperature physical string tension $\sigma(T)$ to emphasize drastic difference in the temperature dependence of $\sigma(T)$ and $\sigma_s(T)$.

Our results for $\sigma_s(T)$ allow to conclude that for temperatures below T_{pc} $\sigma_s(T)$ is constant and equal to σ . This does not come as a surprise since such equality was observed in the quenched theory long ago [16, 17]. Unexpectedly the only previous result for $T < T_{pc}$ in unquenched lattice QCD [5] contradicts our result. In Ref. [5] $\sqrt{\sigma_s(T)}$ for $T < T_{pc}$ is roughly $3T_{pc}$ i.e. essentially higher then $\sqrt{\sigma}$. The reason for such confusing result might be that in Ref. [5] different method to extract the string tension σ_s was used. At $T > T_c$ our results are also different from those of Ref. [5] although agree qualitatively with results of Ref. [8].

Let us now compare our results with two theoretical predictions. In Ref. [21] the following expression for $\sigma_s(T)$ at $T < 2T_{pc}$ was derived within the field correlators approach [23]:

$$\frac{\sigma_s(T)}{\sigma} = \frac{\sinh(M/T) + M/T}{\cosh(M/T) + 1}, \quad (4)$$

where M is the inverse magnetic correlation length and is taken in Ref. [21] equal to 1.5 GeV. Plotting square root of expression (4) in Fig. 2 we took $M/T_c = 7.5$ thus taking $T_c = 200$ MeV. Note that in the considered temperature range the precise value of M/T_c is not important. When $M/T_c \gg 1$ equation (4) suggests very slow increase of the spatial string

tension with temperature in nice agreement with our numerical results. Recently prediction for $\sigma_s(T)$ was derived via AdS/QCD approach [22]:

$$\frac{\sigma_s(T)}{\sigma} = \left(\frac{T}{T_c}\right)^2 \exp \left[\left(\frac{T}{T_c}\right)^2 - 1 \right], \quad T \geq T_c, \quad (5)$$

and $\sigma_s(T) = \sigma$ for $T \leq T_c$. This expression also provide qualitatively good description of our data, as can be seen from Fig. 2.

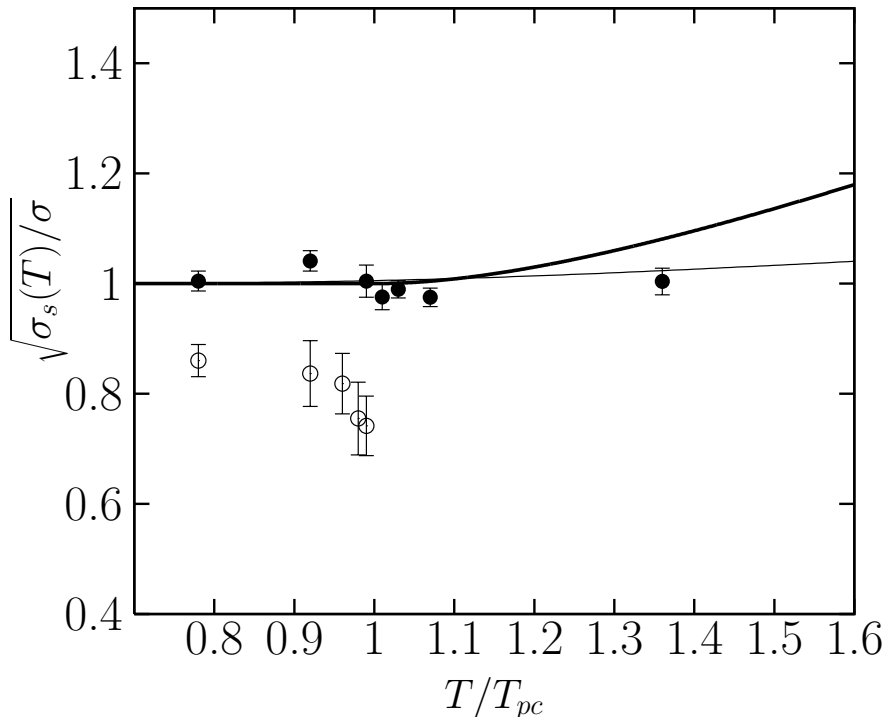


FIG. 2: Square root of the ratio $\sigma_s(T)/\sigma$ (full circles) as function of temperature. The values of σ are shown in Table I, see explanation in the text. Respective data for the finite temperature physical string tension $\sigma(T)$ (empty circles) computed by DIK [20] is shown for comparison. The curves are theoretical predictions from Ref. [21] (thin curve) and Ref. [22] (thick curve).

We shall note that in our computations we do not keep the pion mass fixed, i.e. the computations are not made along the constant physics line. This may introduce sizable systematic effects if σ_s/σ has strong dependence on the quark mass. In Ref. [5] σ_s was computed for two values of m_π/m_ρ , 0.65 and 0.85. The mild dependence on the quark mass was observed. These results need to be confirmed. In any case our conclusion about independence of σ_s on the temperature below transition is intact since $m_\pi r_0$ varies by only

about 10% when temperature increases from $T/T_{pc} = 0.78$ to $T/T_{pc} = 1.01$, as can be seen from the Table I.

Let us summarize our main results. We presented first numerical evidence in unquenched lattice QCD that for temperatures below T_{pc} $\sigma_s(T)$ is constant and equal to σ , the physical string tension at $T = 0$. This is in agreement with quenched lattice QCD results and theoretical predictions. The theoretical predictions eq.(4) [21] and eq(5) [22] for $\sigma_s(T)$ at temperatures $T_{pc} < T < 1.4T_{pc}$ where all known lattice results clearly deviate from the expected high temperature behavior

$$\sqrt{\sigma_s(T)} = cg^2(T)T \quad (6)$$

are in good agreement with our results with better agreement for eq.(4).

Acknowledgments

We wish to thank N.O. Agasian, M.I. Polikarpov and Yu.A. Simonov for useful discussions. This work was completed due to financial support by the Russian Federal Agency of Atomic Energy. It is also partially supported by grants RFBR 05-02-16306, RFBR 07-02-00237, RFBR-DFG 06-02-04010.

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- [24] It is known that at zero temperature the physical string tension σ decreases with decreasing of the quark mass [19]. We took this into account using for σ results of Ref. [19] for $m_\pi r_0$ values close to our respective values as shown in Table I.